Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] state gradient	•[1] $-\frac{5}{2}$	3
			•[2] state perpendicular gradient	•[2] $\frac{2}{5}$	
			•[3] find equation of line	•[3] $5y = 2x + 32$	

Notes:

1. At •[1], ignore any errors in processing the constant term.
2. At •[1] and •[2], ignore the appearance of 'x'.
3. •[3] is only available as a consequence of using a perpendicular gradient.
4. At •[3], accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A A perpendicular gradient has been clearly stated $5x + 2y = 7$ $m_{\perp} = \frac{2}{5}$ •[1] ✓ •[2] ✓ $5y = 2x + 32$ •[3] ✓	**Candidate B** No communication for perpendicular gradient $5x + 2y = 7$ $y = -\frac{5}{2}x + \frac{7}{2}$ $m = \frac{2}{5}$ •[1] ^ •[2] $\boxed{✓ 1}$ $5y = 2x + 32$ •[3] $\boxed{✓ 1}$
Candidate C $m = 5$ •[1] ✗ $m_{\perp} = -\frac{1}{5}$ •[2] $\boxed{✓ 1}$ $x + 5y = 29$ •[3] $\boxed{✓ 1}$	

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] apply $m\log_n x = \log_n x^m$ •[2] apply $\log_n x - \log_n y = \log_n \frac{x}{y}$ •[3] evaluate	•[1] $\log_3 6^2$ •[2] $\log_3 \frac{6^2}{4}$ •[3] 2	3

Notes:

1. Do not penalise the omission of the base of the logarithm at •[1] or •[2].
2. Correct answer with no working, award 0/3.

Commonly Observed Responses:

Candidate A - introducing a variable		**Candidate B**	
$\log_3 9$	•[1] ✓ •[2] ✓	$2\log_3\left(\frac{6}{4}\right)$	•[2] ✗
$3^x = 9$		$\log_3\left(\frac{6}{4}\right)^2$	•[1] $\boxed{✓ 1}$ •[3] ^
$x = 2$	•[3] ✓		

Question			Generic scheme	Illustrative scheme	Max mark
3.			**Method 1** $\bullet^1$ equate composite function to x $\bullet^2$ write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$ $\bullet^3$ state inverse function	**Method 1** $\bullet^1$ $h\left(h^{-1}(x)\right)=x$ $\bullet^2$ $4+\frac{1}{3}h^{-1}(x)=x$ $\bullet^3$ $h^{-1}(x)=3(x-4)$	3
			Method 2 $\bullet^1$ write as $y=h(x)$ and start to rearrange $\bullet^2$ express x in terms of y $\bullet^3$ state inverse function	**Method 2** $\bullet^1$ $y=h(x)\Rightarrow x=h^{-1}(y)$ $y-4=\frac{1}{3}x$ or $3y=12+x$ $\bullet^2$ $x=3(y-4)$ $\bullet^3$ $h^{-1}(y)=3(y-4)$ $\Rightarrow h^{-1}(x)=3(x-4)$	

Notes:

1. In Method 1, accept $4+\frac{1}{3}h^{-1}(x)=x$ for $\bullet^1$ and $\bullet^2$.
2. In Method 2, accept '$y-4=\frac{1}{3}x$' without reference to $y=h(x)\Rightarrow x=h^{-1}(y)$ at $\bullet^1$.
3. In Method 2, accept $h^{-1}(x)=3(x-4)$ without reference to $h^{-1}(y)$ at $\bullet^3$.
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent. See Candidates A and B for example.
5. At $\bullet^3$ stage, accept h^{-1} written in terms of any dummy variable eg $h^{-1}(y)=3(y-4)$.
6. $y=3(x-4)$ does not gain $\bullet^3$.
7. $h^{-1}(x)=3(x-4)$ with no working gains 3/3.

Question		Generic scheme	Illustrative scheme	Max mark
3.	(continued)			

Commonly Observed Responses:

Candidate A $h(x)=4+\frac{1}{3}x$ $y=4+\frac{1}{3}x$ $x=3(y-4)$ $\quad\bullet^1$ ✓ $\bullet^2$ ✓ $y=3(x-4)$ $\quad\bullet^3$ ✗ $h^{-1}(x)=3(x-4)$	**Candidate B** $h(x)=4+\frac{1}{3}x$ $y=4+\frac{1}{3}x$ $x=4+\frac{1}{3}y$ $\quad\bullet^1$ ✗ $y=3(x-4)$ $\quad\bullet^2$ $\boxed{✓ 1}$ $h^{-1}(x)=3(x-4)$ $\quad\bullet^3$ $\boxed{✓ 1}$
Candidate C - BEWARE $h'=\ldots$ $\quad\bullet^3$ ✗	**Candidate D** $x\rightarrow x\div 3\rightarrow x\div 3+4=h(x)$ $\div 3\rightarrow +4$ $\therefore -4\rightarrow \times 3$ $\quad\bullet^1$ ✓ $3(x-4)$ $\quad\bullet^2$ ✓ $h^{-1}(x)=3(x-4)$ $\quad\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
4.			$\bullet^1$ express first term in differentiable form	$\bullet^1$ $y = x^{\frac{3}{2}}\ldots$ stated or implied by $\bullet^2$	3
			$\bullet^2$ differentiate first term	$\bullet^2$ $\frac{3}{2}x^{\frac{1}{2}}\ldots$	
			$\bullet^3$ differentiate second term	$\bullet^3$ $\ldots + 2x^{-2}$	

Notes:

1. $\bullet^2$ is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate throughout, only $\bullet^1$ is available.

Commonly Observed Responses:

Candidate A - differentiating over two lines

$y = x^{\frac{3}{2}} + 2x^{-2}$	$\bullet^1$ ✓
$y = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-2}$	$\bullet^2$ ✓ $\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ use $m = \tan\theta$	•¹ $m = \tan\frac{\pi}{6}$ or $m = \tan 30°$	3
			•² evaluate exact value	•² $\frac{1}{\sqrt{3}}$	
			•³ determine equation	•³ eg $y\sqrt{3} = x + 2$ or $y = \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$	

Notes:

1. Do not award •¹ for $m = \tan^{-1}\frac{\pi}{6}$. However •² and •³ are still available. Where candidates state $m = \tan^{-1}\frac{\pi}{3}$ only •³ is available.
2. Where candidates make no reference to a trigonometric ratio or use an incorrect trigonometric ratio, •¹ and •² are unavailable.
3. •³ is only available as a consequence of attempting to use a tan ratio. See Candidate F
4. Accept $y = \frac{1}{\sqrt{3}}(x+2)$ for •³, but do not accept $y - 0 = \frac{1}{\sqrt{3}}(x+2)$.

Commonly Observed Responses:

Candidate A $m = \tan\frac{\pi}{3}$ •¹ ✗ $m = \sqrt{3}$ •² ✓ 1 $y = \sqrt{3}x + 2\sqrt{3}$ •³ ✓ 1	**Candidate B** $m = \frac{1}{\sqrt{3}}$ (with or without a diagram) •¹ ^ •² ✓ 2 $y = \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$ •³ ✓ 1
Candidate C $m = \tan\theta$ (with or without a diagram) •¹ ^ $m = \frac{1}{\sqrt{3}}$ •² ✓ 1	**Candidate D** $m = \tan\theta$ (with or without a diagram) •¹ ^ $m = \sqrt{3}$ •² ✗ $y = \sqrt{3}x + 2\sqrt{3}$ •³ ✓ 1
Candidate E $m = \tan\theta = \frac{\pi}{6}$ •¹ ✗ $m = \frac{1}{\sqrt{3}}$ •² ✓ 1	**Candidate F** $m = \tan\frac{\pi}{3}$ •¹ ✗ $m = 60$ •² ✗ $y = 60(x+2)$ •³ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			$\bullet^1$ start to integrate	$\bullet^1$ $\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$	4
			$\bullet^2$ complete integration	$\bullet^2$ $\ldots\times -\dfrac{1}{3}$	
			$\bullet^3$ process limits	$\bullet^3$ $-\frac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\frac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$	
			$\bullet^4$ evaluate integral	$\bullet^4$ 2	

Notes:

1. For candidates who **differentiate throughout** or make **no attempt to integrate**, award 0/4.
2. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
3. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after $\bullet^1$.
4. $\bullet^3$ is only available for substitution into an expression which is equivalent to the integrand obtained at $\bullet^2$.
5. The integral obtained must contain a non-integer power for $\bullet^4$ to be available.
6. $\bullet^4$ is only available to candidates who deal with the coefficient of x at the $\bullet^2$ stage. See Candidate A.

Commonly Observed Responses:

Candidate A

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$ $\quad\bullet^1$ ✓ $\bullet^2$ ^

$2(10-3(2))^{\frac{1}{2}}-2(10-3(-5))^{\frac{1}{2}}$ $\quad\bullet^3$ $\boxed{✓ 1}$

-6 $\quad\bullet^4$ $\boxed{✓ 2}$ Note 6

Candidate B - NOT differentiating throughout

$-\frac{1}{2}(10-3x)^{-\frac{3}{2}}\times -\frac{1}{3}$ $\quad\bullet^1$ ✗ $\bullet^2$ ✓

$\frac{1}{6}(10-3(2))^{-\frac{3}{2}}-\frac{1}{6}(10-3(-5))^{-\frac{3}{2}}$ $\quad\bullet^3$ $\boxed{✓ 1}$

$\dfrac{39}{2000}$ $\quad\bullet^4$ $\boxed{✓ 1}$

Candidate C

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times -3$ $\quad\bullet^1$ ✓ $\bullet^2$ ✗

$-6(10-3(2))^{\frac{1}{2}}-\left(-6(10-3(-5))^{\frac{1}{2}}\right)$ $\quad\bullet^3$ $\boxed{✓ 1}$

18 $\quad\bullet^4$ $\boxed{✓ 1}$

Candidate D - integrating over two lines

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times -\dfrac{1}{3}$ $\quad\bullet^1$ ✓ $\bullet^2$ ^

$-\frac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\frac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$ $\quad\bullet^3$ $\boxed{✓ 1}$

2 $\quad\bullet^4$ $\boxed{✓ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)	(i)	•¹ determine $\sin r$	•¹ $\dfrac{1}{\sqrt{10}}$	**1**
		(ii)	•² determine $\sin q$	•² $\dfrac{3}{\sqrt{13}}$	**1**

Notes:

1. In (a)(ii), where candidates do not simplify the perfect square see Candidates A and B.

Commonly Observed Responses:

Candidate A	**Candidate B** - simplification in part (b)
$\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •² $\boxed{\checkmark\ 2}$	(a)(ii) $\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •² ✓ (b) $\sin(q-r) = \dfrac{7}{\ldots}$ Roots have been simplified in (b)

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(b)		•³ select appropriate formula and express in terms of p and q •⁴ substitute into addition formula •⁵ evaluate $\sin(q-r)$	•³ $\sin q\cos r - \cos q\sin r$ stated or implied by •⁴ •⁴ $\frac{3}{\sqrt{13}}\times\frac{3}{\sqrt{10}}-\frac{2}{\sqrt{13}}\times\frac{1}{\sqrt{10}}$ •⁵ $\frac{7}{\sqrt{130}}$	**3**

Notes:

2. Award •³ for candidates who write $\sin\left(\frac{3}{\sqrt{13}}\right)\times\cos\left(\frac{3}{\sqrt{10}}\right)-\sin\left(\frac{2}{\sqrt{13}}\right)\times\cos\left(\frac{1}{\sqrt{10}}\right)$. •⁴ and •⁵ are unavailable.
3. For any attempt to use $\sin(q-r)=\sin q-\sin r$, •⁴ and •⁵ are unavailable.
4. At •⁵, the answer must be given as a single fraction. Accept $\frac{7}{\sqrt{13}\sqrt{10}}$, $\frac{7\sqrt{10}}{10\sqrt{13}}$ and $\frac{7\sqrt{13}}{13\sqrt{10}}$.
5. Do not penalise trigonometric ratios which are less than –1 or greater than 1.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			**Method 1**	**Method 1**	4
			•¹ apply $\log_6 x + \log_6 y = \log_6 xy$	•¹ $\log_6\left(x(x+5)\right) = \ldots$	
			•² write in exponential form	•² $x(x+5) = 6^2$	
			•³ express in standard quadratic form	•³ $x^2 + 5x - 36 = 0$	
			•⁴ solve quadratic and state solution of log equation	•⁴ -9, 4 and $x > 0 \Rightarrow x = 4$	
			Method 2	**Method 2**	
			•¹ apply $\log_6 x + \log_6 y = \log_6 xy$	•¹ $\log_6\left(x(x+5)\right) = \ldots$	
			•² apply $m\log_6 x = \log_6 x^m$	•² $\ldots = \log_6 6^2$	
			•³ express in standard quadratic form	•³ $x^2 + 5x - 36 = 0$	
			•⁴ solve quadratic and state solution of log equation	•⁴ -9, 4 and $x > 0 \Rightarrow x = 4$	

Notes:

1. Accept $\log_6 x(x+5) = \ldots$ for $\bullet^1$.
2. $\bullet^2$ is not available for $x(x+5) = 2^6$; however candidates may still gain $\bullet^3$ and $\bullet^4$.
3. $\bullet^3$ and $\bullet^4$ are only available if the quadratic reached at $\bullet^3$ is obtained by applying the rules in $\bullet^1$ and $\bullet^2$.
4. $\bullet^4$ is only available for solving a polynomial of degree two or higher.
5. At $\bullet^4$, accept any indication that -9 has been discarded. For example, scoring out $x = -9$ or underlining $x = 4$.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_6\left(x(x+5)\right) = 2$	$\bullet^1$ ✓	$\log_6\left(x(x+5)\right) = 2$	$\bullet^1$ ✓
$x(x+5) = 12$	$\bullet^2$ ✗	$x(x+5) = 64$	$\bullet^2$ ✗
$x^2 + 5x - 12 = 0$	$\bullet^3$ $\boxed{✓\ 1}$	$x^2 + 5x - 64 = 0$	$\bullet^3$ $\boxed{✓\ 1}$
$\frac{-5 \pm \sqrt{73}}{2}$ and $x > 0 \Rightarrow x = \frac{-5 + \sqrt{73}}{2}$	$\bullet^4$ $\boxed{✓\ 1}$	$\frac{-5 \pm \sqrt{281}}{2}$ and $x > 0 \Rightarrow x = \frac{-5 + \sqrt{281}}{2}$	$\bullet^4$ $\boxed{✓\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mar
9.			$\bullet^1$ substitute for $\cos 2x^\circ$ into equation	$\bullet^1$ $2\cos^2 x^\circ - 1\ldots$	**5**
			$\bullet^2$ express in standard quadratic form	$\bullet^2$ $2\cos^2 x^\circ - 5\cos x^\circ + 2 = 0$	
			$\bullet^3$ factorise	$\bullet^3$ $(2\cos x^\circ - 1)(\cos x^\circ - 2) = 0$	
				$\bullet^4$ $\bullet^5$	
			$\bullet^4$ solve for $\cos x^\circ$	$\bullet^4$ $\cos x^\circ = \frac{1}{2}$ $\cos x^\circ = 2$	
			$\bullet^5$ solve for x	$\bullet^5$ $x = 60,\ 300$ 'no solutions'	

Notes:

1. $\bullet^1$ is not available for simply stating $\cos 2x^\circ = 2\cos^2 x^\circ - 1$ with no further working.
2. In the event of $\cos^2 x^\circ - \sin^2 x^\circ$ or $1 - 2\sin^2 x^\circ$ being substituted for $\cos 2x^\circ$, $\bullet^1$ cannot be awarded until the equation reduces to a quadratic in $\cos x^\circ$.
3. Substituting $2\cos^2 \mathrm{A} - 1$ or $2\cos^2 \alpha - 1$ for $\cos 2x^\circ$ at the $\bullet^1$ stage should be treated as bad form provided the equation is written in terms of x at $\bullet^2$ stage. Otherwise, $\bullet^1$ is not available.
4. Do not penalise the omission of degree signs.
5. '$=0$' must appear by $\bullet^3$ stage for $\bullet^2$ to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at $\bullet^2$ stage for $\bullet^2$ to be awarded.
6. $\cos x^\circ = \dfrac{5 \pm \sqrt{9}}{4}$ gains $\bullet^3$.
7. Candidates may express the equation obtained at $\bullet^2$ in the form $2c^2 - 5c + 2 = 0$ or $2x^2 - 5x + 2 = 0$. In these cases, award $\bullet^3$ for $(2c-1)(c-2) = 0$ or $(2x-1)(x-2) = 0$.
   However, $\bullet^4$ is only available if $\cos x^\circ$ appears explicitly at this stage. See Candidate A.
8. The equation $2 + 2\cos^2 x^\circ - 5\cos x^\circ = 0$ does not gain $\bullet^2$ unless $\bullet^3$ has been awarded.
9. $\bullet^4$ and $\bullet^5$ are only available as a consequence of trying to solve a quadratic equation. See Candidate B. However, $\bullet^5$ is not available if the quadratic equation has repeated roots.
10. $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$. See Candidate C.
11. $\bullet^5$ is only available for 2 valid solutions within the stated range. Ignore 'solutions' outwith the range. However, see Candidate E.
12. Accept $\cancel{\cos x^\circ = 2}$ for $\bullet^5$. See Candidate A.

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.	(continued)		

Commonly Observed Responses:

Candidate A

$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$ $\quad \bullet^1$ ✓

$2c^2 - 5c + 2 = 0$ $\quad \bullet^2$ ✓

$(2c-1)(c-2) = 0$ $\quad \bullet^3$ ✓

$c = \frac{1}{2},\ c = 2$ $\quad \bullet^4$ ✗

$x = 60,\ 300 \quad \cancel{\cos x^\circ = 2}$ $\quad \bullet^5$ $\boxed{\checkmark\ 1}$

Candidate B - not solving a quadratic

$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$ $\quad \bullet^1$ ✓

$2\cos^2 x^\circ - 5\cos x^\circ + 2 = 0$ $\quad \bullet^2$ ✓

$-3\cos x^\circ + 2 = 0$ $\quad \bullet^3$ ✗

$\cos x^\circ = \frac{2}{3}$ $\quad \bullet^4$ $\boxed{\checkmark\ 2}$ $\bullet^5$ ^

Candidate C - not in standard quadratic form

$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$ $\quad \bullet^1$ ✓

$2\cos^2 x^\circ - 5\cos x^\circ = -2$ $\quad \bullet^2$ $\boxed{\checkmark\ 2}$

$\cos x^\circ(2\cos x^\circ - 5) = -2$ $\quad \bullet^3$ $\boxed{\checkmark\ 2}$

$\cos x^\circ = -2,\ 2\cos x^\circ - 5 = -2$

$\Rightarrow \cos x = \frac{3}{2}$ $\quad \bullet^4$ ✗

No solutions $\quad \bullet^5$ ✗

Candidate D - reading $\cos 2x^\circ$ as $\cos^2 x^\circ$

$\cos^2 x^\circ = 5\cos x^\circ - 3$ $\quad \bullet^1$ ✗

$\cos^2 x^\circ - 5\cos x^\circ + 3 = 0$ $\quad \bullet^2$ $\boxed{\checkmark\ 1}$

$\cos x^\circ = \frac{5 \pm \sqrt{13}}{2}$ $\quad \bullet^3$ $\boxed{\checkmark\ 1}$

$\bullet^4$ ^ $\bullet^5$ ^

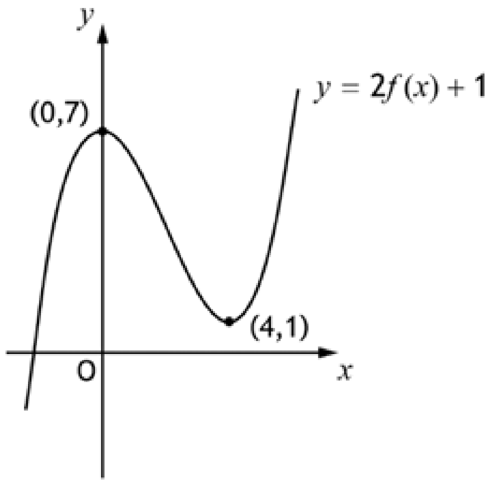
Candidate E

⋮ $\quad \bullet^1$ ✓ $\bullet^2$ ✓

$(\cos x^\circ - 1)(\cos x^\circ - 2) = 0$ $\quad \bullet^3$ ✗

$\cos x^\circ = 1, \qquad \cos x^\circ = 2$ $\quad \bullet^4$ $\boxed{\checkmark\ 1}$

$x = 0 \qquad$ No solutions $\quad \bullet^5$ $\boxed{\checkmark\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		•¹ vertical scaling by a factor of 2 identifiable from graph •² vertical translation of '+1' units identifiable from graph •³ transformations applied in correct order	•¹ •² •³ 	**3**

Notes:

1. •¹, •² and •³ are only available for a 'cubic' with a maximum and minimum turning point.
2. Ignore intersections (or lack of intersections) with the original graph.

Commonly Observed Responses:

Where the image of (4,0) is not (4,1), that point must be annotated (or drawn to within tolerance).
In the following table, the images of the given points must be stationary points for the marks to be awarded.

Image of (0,3)	Image of (4,0)	Award…	
(0,8)	(4,2)	2/3	Transformation in wrong order
(0,4) (0,4) (0,4)	(8,1) (4,1) (2,1)	1/3 1/3 1/3	Only vertical translation correct
(0,5)	(4,−1)	2/3	Evidence of vertical scaling and transformation in correct order
(0,6) (0,7) (1,6) (−1,6)	(4,0) any incorrect point (5,0) (3,0)	1/3 1/3 1/3 1/3	Evidence of vertical scaling
(0,−2) (0,4)	(4,1) (−4,1)	1/3 1/3	Evidence of vertical translation
(0,5) (0,2)	any other point any other point	0/3 0/3	Insufficient evidence of scaling/translation

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(b)		$\bullet^4$ state coordinates of stationary points	$\bullet^4$ $(0,3)$ and $(8,0)$	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.			**Method 1** •[1] identify common factor •[2] complete the square •[3] process for r and write in required form	**Method 1** •[1] $2(x^2+6x\ldots$ stated or implied by •[2] •[2] $2(x+3)^2\ldots$ •[3] $2(x+3)^2+5$	3
			Method 2 •[1] expand completed square form •[2] equate coefficients •[3] process for q and r and write in required form	**Method 2** •[1] $px^2+2pqx+pq^2+r$ •[2] $p=2,\ 2pq=12,\ pq^2+r=23$ •[3] $2(x+3)^2+5$	

Notes:

1. $2(x+3)^2+5$ with no working gains •[1] and •[2] only. However, see Candidate E.

Commonly Observed Responses:

Candidate A	Candidate B
$2(x^2+6)+23$ $2((x+3)^2-9)+23$ •[1] ✓ •[2] ✓ $2(x+3)^2+5$ •[3] ✓ See the exception to marking principle (h)	$px^2+2pqx+pq^2+r$ •[1] ✓ $p=2,\ 2pq=12,\ pq^2+r=23$ •[2] ✓ $q=3,\ r=5$ •[3] ^ •[3] is lost as answer is not in completed square form
Candidate C $2(x^2+12x)+23$ •[1] ✗ $2((x+6)^2-36)+23$ •[2] $\boxed{✓\ 1}$ $2(x+6)^2-49$ •[3] $\boxed{✓\ 1}$	**Candidate D** $2((x+6)^2-36)+23$ •[1] ✗ •[2] ✗ $2(x+6)^2-49$ •[3] $\boxed{✓\ 1}$
Candidate E $2(x+3)^2+5$ •[1] ✓ •[2] ✓ Check: $=2(x^2+6x+9)+5$ $=2x^2+12x+18+5$ $=2x^2+12x+23$ •[3] ✓	

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.			•[1] start to differentiate •[2] complete differentiation •[3] evaluate derivative	•[1] $4\cos\left(3x-\frac{\pi}{3}\right)\ldots$ •[2] $\ldots\times 3$ •[3] $6\sqrt{3}$	3

Notes:

1. Where candidates make no attempt to differentiate or use another invalid approach, •[2] and •[3] are not available.
2. At the •[1] and •[2] stage, candidates who work in degrees cannot gain •[1]. However •[2] and •[3] are still available.
3. At the •[3] stage, do not penalise candidates who work in degrees or in radians and degrees.
4. Ignore the appearance of $+c$ at any stage.

Commonly Observed Responses:

Candidate A **Differentiating over two lines** $f'(x)=4\cos\left(3x-\frac{\pi}{3}\right)$ •[1] ✓ $f'(x)=12\cos\left(3x-\frac{\pi}{3}\right)$ •[2] ^ $6\sqrt{3}$ •[3] ✓ 1	**Candidate B** $4\cos\left(3x-\frac{\pi}{3}\right)\times\frac{1}{3}$ •[1] ✓ •[2] ✗ $\frac{2\sqrt{3}}{3}$ •[3] ✓ 1	**Candidate C** $4\cos\left(3x-\frac{\pi}{3}\right)$ •[1] ✓ •[2] ^ $2\sqrt{3}$ •[3] ✓ 1
Candidate D $\pm 12\sin\left(3x-\frac{\pi}{3}\right)$ •[1] ✗ •[2] ✗ ± 6 •[3] ✓ 1	**Candidate E** $\pm 4\sin\left(3x-\frac{\pi}{3}\right)\ldots$ •[1] ✗ $\ldots\times 3$ •[2] ✓ 1 ± 6 •[3] ✓ 1	**Candidate F** $-12\cos\left(3x-\frac{\pi}{3}\right)$ •[1] ✗ •[2] ✓ $-6\sqrt{3}$ •[3] ✓ 1

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)	(i)	$\bullet^1$ use -2 in synthetic division or evaluation of the cubic $\bullet^2$ complete division/evaluation and interpret result	$\bullet^1$ $\begin{array}{r\|rrrr} -2 & 1 & -2 & -20 & -24 \\ & & & & \\ \hline & 1 & & & \end{array}$ or $(-2)^3-2(-2)^2-20(-2)-24$ $\bullet^2$ $\begin{array}{r\|rrrr} -2 & 1 & -2 & -20 & -24 \\ & & -2 & 8 & 24 \\ \hline & 1 & -4 & -12 & \boxed{0} \end{array}$ Remainder $=0 \therefore (x+2)$ is a factor or $f(-2)=0 \therefore (x+2)$ is a factor	**2**
		(ii)	$\bullet^3$ state quadratic factor $\bullet^4$ find remaining factors or apply the quadratic formula $\bullet^5$ state solution	$\bullet^3$ $x^2-4x-12$ $\bullet^4$ $(x+2)$ and $(x-6)$ or $\dfrac{4\pm\sqrt{(-4)^2-4(1)(-12)}}{2(1)}$ $\bullet^5$ $-2, 6$	**3**

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage - a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(-2)=0$ so $(x+2)$ is a factor'
   - 'since remainder $=0$, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore, \rightarrow, \Rightarrow$ etc.
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-2$ is a factor', '... is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

	(b)		$\bullet^6$ state value of k	$\bullet^6$ 3	**1**

Notes:

1. Accept $y=f(x-3)$ or $f(x-3)$ for $\bullet^6$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)	(i)	•¹ state coordinates of centre •² state radius	•¹ $(7,-5)$ •² 10	2
		(ii)	•³ substitute coordinates of P and evaluate •⁴ communicate result	•³ $(-2-7)^2+(7+5)^2=225$ •⁴ $225>100 \therefore$ P lies outside	2

Notes:

1. Accept $x=7,\ y=-5$ for •¹.
2. Do not accept $g=7,\ f=-5$ or $7,\ -5$ for •¹.

Commonly Observed Responses:

Candidate A $d=15$ •³ ✓ $15>10 \therefore$ P lies outside •⁴ ✓	**Candidate B** $d=\sqrt{225}$ •³ ✓ $\sqrt{225}>10 \therefore$ P lies outside •⁴ ✓
Candidate C $d=15$ •³ ✓ $r=10$ $d>r \therefore$ P lies outside •⁴ ✓	

	(b)		•⁵ determine first value of r •⁶ determine second value of r	•⁵ 5 •⁶ 25	2

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ determine gradient of AB $\bullet^2$ determine gradient of altitude $\bullet^3$ find equation	$\bullet^1$ -1 $\bullet^2$ 1 $\bullet^3$ $y = x - 4$	**3**

Notes:

1. $\bullet^3$ is only available to candidates who find and use a perpendicular gradient.
2. At $\bullet^3$, accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A - BEWARE

Correct gradient from incorrect substitution

$m_{AB} = \dfrac{2-(-1)}{-4-(-1)} = -1$ $\bullet^1$ ✘

$m_{\perp} = 1$ $\bullet^2$ $\boxed{✓ 1}$

$y = x - 4$ $\bullet^3$ $\boxed{✓ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		•4 determine midpoint of AC •5 determine gradient of median •6 find equation	•4 $(3,1)$ •5 5 •6 $y=5x-14$	**3**

Notes:

3. •5 is only available to candidates who use a midpoint to find a gradient.
4. •6 is only available as a consequence of using a 'midpoint' of AC and the point B.
5. At •6, accept any arrangement of a candidate's equation where constant terms have been simplified.
6. •6 is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular bisector of AC		Candidate B - Altitude through B	
$\text{Midpoint}_{AC}\ (3,1)$	•1 ✓	$m_{AC}=\frac{1}{2}$	•1 ^
$m_{AC}=\frac{1}{2}\Rightarrow m_{\perp}=-2$	•2 ✗	$m_{\perp}=-2$	•2 ✗
$y+2x=7$	•3 $\boxed{✓\ 2}$	$y+2x=0$	•3 $\boxed{✓\ 2}$
For other perpendicular bisectors award 0/3			

Candidate C - Median through A		Candidate D - Median through C	
$\text{Midpoint}_{BC}\left(\frac{9}{2},-\frac{1}{2}\right)$	•1 ✗	$\text{Midpoint}_{AB}\left(\frac{1}{2},-\frac{5}{2}\right)$	•1 ✗
$m_{AM}=\frac{1}{11}$	•2 $\boxed{✓\ 1}$	$m_{CM}=\frac{11}{13}$	•2 $\boxed{✓\ 1}$
$11y=x-10$	•3 $\boxed{✓\ 2}$	$13y=11x-38$	•3 $\boxed{✓\ 2}$

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•7 determine x-coordinate •8 determine y-coordinate	•7 2.5 •8 -1.5	**2**

Notes:

7. For $\left(\frac{10}{4},-\frac{6}{4}\right)$ award 1/2 (do not penalise repeated lack of simplification - *general marking principle* (*l*)).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ use discriminant $\bullet^2$ apply condition and simplify $\bullet^3$ state range	$\bullet^1$ $(-8)^2 - 4(2)(4-p)$ $\bullet^2$ $32+8p>0$ or $8p>-32$ $\bullet^3$ $p>-4$	3

Notes:

1. At $\bullet^1$, treat the inconsistent use of brackets eg $(-8)^2 - 4\times2\times4-p$ or $-8^2-4(2)(4-p)$ as bad form only if the candidate deals with the unbracketed terms correctly in the next line of working.
2. If candidates have the condition 'discriminant $=0$', then $\bullet^2$ and $\bullet^3$ are unavailable. However, see Candidate E.
3. If candidates have the condition 'discriminant <0', 'discriminant ≤ 0' or 'discriminant ≥ 0' then $\bullet^2$ is lost but $\bullet^3$ is available.

Commonly Observed Responses:

Candidate A - bad form $(-8)^2-4\times2\times\underline{4-p}>0$ $32+8p>0$ $\quad\bullet^1 \checkmark \;\bullet^2 \checkmark$ $p>-4$ $\quad\bullet^3 \checkmark$	**Candidate B - no coefficient of p** $(-8)^2-4\times2\times\underline{4-p}>0$ $32-p>0$ $\quad\bullet^1 \times \;\bullet^2 \boxed{\checkmark 2}$ $p<32$ $\quad\bullet^3 \boxed{\checkmark 2}$
Candidate C - bad form $\underline{-8^2}-4\times2\times(4-p)>0$ $32+8p>0$ $\quad\bullet^1 \checkmark \;\bullet^2 \checkmark$ $p>-4$ $\quad\bullet^3 \checkmark$	**Candidate D - not bad form** $\underline{-8^2}-4\times2\times(4-p)>0$ $-96+8p>0$ $\quad\bullet^1 \times \;\bullet^2 \boxed{\checkmark 2}$ $p>12$ $\quad\bullet^3 \boxed{\checkmark 1}$
Candidate E - condition stated initially Real and distinct roots $b^2-4ac>0$ $(-8)^2-4(2)(4-p)=0$ $\quad\bullet^1 \checkmark$ $32+8p=0$ $p=-4$ so $p>-4$ $\quad\bullet^2 \checkmark \;\bullet^3 \checkmark$	**Candidate F** $8^2-4(2)(4-p)>0$ $\quad\bullet^1 \times$ $32+8p>0$ $\quad\bullet^2 \boxed{\checkmark 1}$ $p>-4$ $\quad\bullet^3 \boxed{\checkmark 1}$ However, $64-4(2)(4-p)>0$ as the first line of working may be awarded $\bullet^1$

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•¹ use compound angle formula •² compare coefficients •³ process for k •⁴ process for a and express in required form	•¹ $k\sin x\cos a + k\cos x\sin a$ stated explicitly •² $k\cos a = 4$ and $k\sin a = 5$ stated explicitly •³ $k = \sqrt{41}$ •⁴ $\sqrt{41}\sin(x + 0.896\ldots)$	4

Notes:

1. Accept $k(\sin x\cos a + \cos x\sin a)$ at $\bullet^1$.
2. Treat $k\sin x\cos a + \cos x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
3. $\sqrt{41}\sin x\cos a + \sqrt{41}\cos x\sin a$ or $\sqrt{41}(\sin x\cos a + \cos x\sin a)$ are acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x = 4$ and $k\sin x = 5$, however $\bullet^4$ may still be gained. See Candidate E.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is not available for a value of a given in degrees.
7. Accept values of a which round to 0.9.
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\sin(x + a)$.
9. Evidence for $\bullet^4$ may not appear until part (b) and must appear by the $\bullet^5$ stage.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$\bullet^1$ ^	$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓	$\sin x\cos a + \cos x\sin a$ $\bullet^1$ ✗
$\sqrt{41}\cos a = 4$ $\sqrt{41}\sin a = 5$ $\bullet^2$ ✓ $\bullet^3$ ✓	$\cos a = 4$ $\sin a = 5$ $\bullet^2$ ✗	$\cos a = 4$ $\sin a = 5$ $\bullet^2$ $\boxed{✓ 2}$
		$k = \sqrt{41}$ $\bullet^3$ ✓
$\tan a = \frac{5}{4}$ $a = 0.896\ldots$	$\tan a = \frac{5}{4}$ $a = 0.896\ldots$ Not consistent with equations at $\bullet^2$.	$\tan a = \frac{5}{4}$ $a = 0.896\ldots$
$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^4$ ✓	$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✗	$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)	**(continued)**			

Commonly Observed Responses:

Candidate D - errors at $\bullet^2$	**Candidate E - use of x at $\bullet^2$**	**Candidate F**
$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓	$k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓	$k\sin A\cos B + k\cos A\sin B$ $\bullet^1$ ✗
$k\cos a = 5$	$k\cos x = 4$	$k\cos A = 4$
$k\sin a = 4$ $\bullet^2$ ✗	$k\sin x = 5$ $\bullet^2$ ✗	$k\sin A = 5$ $\bullet^2$ ✗
$\tan a = \frac{4}{5}$	$\tan x = \frac{5}{4}$	$\tan A = \frac{5}{4}$
$a = 0.674\ldots$	$x = 0.896\ldots$	$A = 0.896\ldots$
$\sqrt{41}\sin(x + 0.674\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$	$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$	$\sqrt{41}\sin(x + 0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^5$ link to (a)	$\bullet^5$ $\sqrt{41}\sin(x + 0.896\ldots) = 5.5$	3
			$\bullet^6$ solve for $(x + a)$	$\bullet^6$ $\overset{\bullet^6}{1.033\ldots}$, $\overset{\bullet^7}{2.108\ldots}$	
			$\bullet^7$ solve for x	$\bullet^7$ $0.137\ldots$, $1.212\ldots$	

Notes:

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. $\bullet^7$ is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At $\bullet^7$ accept values of x which round to 0.1 or 1.2

Commonly Observed Responses:

Candidate G - converting to radians	**Candidate H - working in degrees and truncation**
⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓	⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓
$\sqrt{41}\sin(x + 51.3\ldots)$ $\bullet^4$ ✗	$\sqrt{41}\sin(x + 51.3)$ $\bullet^4$ ✗
$\sqrt{41}\sin(x + 51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{\checkmark\ 1}$	$\sqrt{41}\sin(x + 51.3) = 5.5$ $\bullet^5$ $\boxed{\checkmark\ 1}$
$x + 51.3\ldots = 59.1\ldots, 120.8\ldots$	$x + 51.3 = 59.1, 120.9$
$x = 7.8\ldots, 69.4\ldots$ $\bullet^6$ $\boxed{\checkmark\ 1}$	$x = 7.8, 69.6$ $\bullet^6$ $\boxed{\checkmark\ 1}$ $\bullet^7$ ^
$x = \frac{7.9\pi}{180}, \frac{69.5\pi}{180}$ $\bullet^7$ $\boxed{\checkmark\ 1}$	

Candidate I - working in degrees	**Candidate J - working in degrees**
⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓	⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓
$\sqrt{41}\sin(x + 51.3\ldots)$ $\bullet^4$ ✗	$\sqrt{41}\sin(x + 51.3\ldots)$ $\bullet^4$ ✗
$\sqrt{41}\sin(x + 51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{\checkmark\ 1}$	$\sqrt{41}\sin(x + 51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{\checkmark\ 1}$
$x + 51.3\ldots = 59.1\ldots$	$x + 51.3\ldots = 59.1\ldots, 120.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^
$x = 7.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^	

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ state appropriate integral	•¹ $\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)dx$	4
			•² integrate	•² $\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$	
			•³ substitute limits	•³ $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$ $-\left(\frac{1}{4}(-1)^4-\frac{5}{3}(-1)^3+(-1)^2+8(-1)\right)$	
			•⁴ evaluate area	•⁴ $\frac{63}{4}$ or 15.75	

Notes:

1. Limits and 'dx' must appear at the •¹ stage for •¹ to be awarded.
2. Where a candidate differentiates one or more terms at •², then •³ and •⁴ are not available.
3. Candidates who substitute limits without integrating, do not gain •³ or •⁴.
4. Do not penalise the inclusion of '$+c$'.
5. Do not penalise the continued appearance of the integral sign after •¹.
6. •⁴ is not available where solutions include statements such as $-\frac{63}{4}=\frac{63}{4}$. See Candidate C.

Commonly Observed Responses:

Candidate A

$\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)$ •¹ ✗

$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ •² ✓

•³ ^

$=\frac{63}{4}$ •⁴ $\boxed{✓\ 1}$

Candidate B - evidence of substitution using a calculator

$\int\left(x^3-5x^2+2x+8\right)dx$ •¹ ✗

$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ •² ✓

$=\frac{32}{3}-\left(-\frac{61}{12}\right)$ •³ ✓

$=\frac{63}{4}$ •⁴ ✓

Candidate C - communication for •⁴

$\int_{2}^{-1}\left(x^3-5x^2+2x+8\right)dx$ •¹ ✓

... •² ✓ •³ ✓

$=-\frac{63}{4}$, hence area is $\frac{63}{4}$. •⁴ ✓

However $-\frac{63}{4}=\frac{63}{4}$ square units does not gain •⁴

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)		**Method 1**	**Method 1**	3
			•5 state appropriate integral	•5 $\int_2^4 \left(x^3-5x^2+2x+8\right)dx$	
			•6 evaluate integral	•6 $-\frac{16}{3}$	
			•7 interpret result and evaluate total area	•7 $\frac{253}{12}$ or 21.083…	
			Method 2	**Method 2**	
			•5 state appropriate integral	•5 $\int_2^4 \left(0-\left(x^3-5x^2+2x+8\right)\right)dx$	
			•6 substitute limits	•6 $-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)\right)$	
			•7 evaluate total area	•7 $\frac{253}{12}$ or 21.083…	

Notes:

7. For candidates who only consider $\int_{-1}^{4} \ldots dx$ or any other invalid integral, award 0/3.
8. In part (b), at •5 do not penalise the omission of 'dx'.
9. In Method 1, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_2^4$

   or $\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$.
10. In Method 2, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_4^2$ or •5 and •6 may be awarded for

    $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)$.
11. •7 is not available to candidates where solutions include statements such as $-\frac{16}{3}=\frac{16}{3}$ square units. See Candidate D.
12. In Method 1, where a candidate's integral leads to a positive value, •7 is not available.
13. Where a candidate has differentiated in both parts of the question see Candidate E.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)	(continued)			

Commonly Observed Responses:

Candidate D - communication for •7

$\int_2^4 (x^3 - 5x^2 + 2x + 8)\,dx = -\frac{16}{3}$ •5 ✓ •6 ✓

$\frac{63}{4} + \frac{16}{3} = \frac{253}{12}$ •7 ✓

However, •7 is not available where statements such as "$-\frac{16}{3} = \frac{16}{3}$ square units" or "ignore negative" appear.

Candidate E - differentiation in (a) and (b)

(a) $\int_{-1}^{2} (x^3 - 5x^2 + 2x + 8)\,dx$ •1 ✓

$= 3x^2 - 10x + 2$ •2 ✘

$= \left(3(2)^2 - 10(2) + 2\right) - \left(3(-1)^2 - 10(-1) + 2\right)$ •3 ✘

$= -21$

Area $= 21$ •4 ✘

(b) $\left(3(4)^2 - 10(4) + 2\right) - \left(3(2)^2 - 10(2) + 2\right) = 16$ •5 ✓ •6 $\boxed{✓ 1}$

Total Area $= 5$ •7 $\boxed{✓ 2}$ see note 12

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>5.</td><td>(a)</td><td>(i)</td><td>•[1] interpret notation

•[2] state expression for $f\left(g\left(x\right)\right)$</td><td>•[1] $f\left(3x+5\right)$ or $\left(g\left(x\right)\right)^2-2$

•[2] $\left(3x+5\right)^2-2$</td><td>2</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•[3] state expression for $g\left(f\left(x\right)\right)$</td><td>•[3] $3\left(x^2-2\right)+5$</td><td>1</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. For $f\left(g\left(x\right)\right)=\left(3x+5\right)^2-2$ without working, award both •[1] and •[2].</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6">Candidate A
(a)(i) $f\left(g\left(x\right)\right)=3\left(x^2-2\right)+5$ •[1] ✗ •[2] $\boxed{✓ 1}$
(a)(ii) $g\left(f\left(x\right)\right)=\left(3x+5\right)^2-2$ •[3] $\boxed{✓ 1}$</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		$\bullet^4$ interpret information and expand $\bullet^5$ express inequality in standard quadratic form $\bullet^6$ determine zeros of quadratic equation $\bullet^7$ state range with justification	$\bullet^4$ $9x^2+30x+25-2<3x^2-6+5$ $\bullet^5$ $6x^2+30x+24<0$ $\bullet^6$ $-4,\ -1$ $\bullet^7$ $-4<x<-1$ with eg sketch or table of signs	4

Notes:

2. Candidates who do not work with an inequation from the outset lose $\bullet^4$, $\bullet^5$ and $\bullet^7$. However, $\bullet^6$ is still available. See Candidate D.
3. Accept the appearance of $-4,-1$ within inequalities for $\bullet^6$.
4. At $\bullet^7$ accept "$x>-4$ and $x<-1$" or "$x>-4$, $x<-1$" together with the required justification.

Commonly Observed Responses:

Candidate B $9x^2+30x+25-2<3x^2-6+5$ $\quad\bullet^4$ ✓ $6x^2+30x+24<0$ $\quad\bullet^5$ ✓ $6x^2+30x+24=0$ $x=-1,\ x=-4$ $\quad\bullet^6$ ✓ $-4<x<-1$ with sketch $\quad\bullet^7$ ✓	**Candidate C** $9x^2+30x+25-2<3x^2-6+5$ $\quad\bullet^4$ ✓ $6x^2+30x+24=0$ $\quad\bullet^5$ ✗ $x=-1,\ x=-4$ $\quad\bullet^6$ ✓ $-4<x<-1$ with sketch $\quad\bullet^7$ ✗
Candidate D $9x^2+30x+25-2=3x^2-6+5$ $\quad\bullet^4$ ✗ $6x^2+30x+24=0$ $\quad\bullet^5$ ✗ $x=-1,\ x=-4$ $\quad\bullet^6$ ✓ For $f(g(x))<g(f(x))$ $-4<x<-1$ with sketch $\quad\bullet^7$ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ write in integrable form	•¹ $1-3x^{-2}$	5
			•² integrate one term	•² x or $\dots-\frac{3x^{-1}}{-1}$	
			•³ complete integration	•³ $\dots-\frac{3x^{-1}}{-1}+c$ or $x\dots+c$	
			•⁴ interpret information given and substitute for x and y	•⁴ $6=3+3(3)^{-1}+c$	
			•⁵ state expression for y	•⁵ $y=x+3x^{-1}+2$	

Notes:

1. For candidates who make no attempt to integrate only •¹ is available.
2. For candidates who omit $+c$ only •¹ and •² are available.
3. For candidates who differentiate either term, •³, •⁴, and •⁵ are not available.

Commonly Observed Responses:

Candidate A - incomplete substitution		Candidate B - partial integration	
$y=x+3x^{-1}+c$	•¹ ✓ •² ✓ •³ ✓	$y=1+3x^{-1}+c$	•¹ ✓ •² ✓ •³ ✗
$y=3+3(3)^{-1}+c$		$6=1+3(3)^{-1}+c$	•⁴ $\boxed{✓ 1}$
$c=-4$	•⁴ ^	$c=4$	
$y=x+3x^{-1}-4$	•⁵ $\boxed{✓ 1}$	$y=x+3x^{-1}+4$	•⁵ $\boxed{✓ 1}$

Candidate C - inconsistent working		Candidate D - inconsistent working	
$\frac{dy}{dx}=1-\frac{3}{x^2}$		$\frac{dy}{dx}=1-\frac{3}{x^2}$	
$x-3x^{-2}$	•¹ ✗	$x-3x^{-2}$	•¹ ✗
$y=x-\frac{3x^{-1}}{-1}+c$	•² $\boxed{✓ 1}$ •³ $\boxed{✓ 1}$	$y=\frac{x^2}{2}-\frac{3x^{-1}}{-1}+c$	•² $\boxed{✓ 1}$ •³ $\boxed{✓ 1}$

Candidate E integration not complete at •³ stage	
$\frac{dy}{dx}=1-3x^{-2}$	•¹ ✓
$y=x-\frac{3x^{-1}}{-1}$	•² ✓ •³ ✗
$y=x+3x^{-1}+c$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1**	**Method 1**	5
			•1 state equation of line	•1 $\log_5 y = -2\log_5 x + 3$	
			•2 introduce logs	•2 $\log_5 y = -2\log_5 x + 3\log_5 5$	
			•3 use laws of logs	•3 $\log_5 y = \log_5 x^{-2} + \log_5 5^3$	
			•4 use laws of logs	•4 $\log_5 y = \log_5 5^3 x^{-2}$	
			•5 state k and n	•5 $k = 125,\ n = -2$	
			Method 2	**Method 2**	
			•1 state equation of line	•1 $\log_5 y = -2\log_5 x + 3$	
			•2 use laws of logs	•2 $\log_5 y = \log_5 x^{-2} + 3$	
			•3 use laws of logs	•3 $\log_5 \frac{y}{x^{-2}} = 3$	
			•4 use laws of logs	•4 $\frac{y}{x^{-2}} = 5^3$	
			•5 state k and n	•5 $k = 125, n = -2$	
			Method 3	**Method 3** **The equations at •1, •2, and •3 must be stated explicitly.**	
			•1 introduce logs to $y = kx^n$	•1 $\log_5 y = \log_5 kx^n$	
			•2 use laws of logs	•2 $\log_5 y = n\log_5 x + \log_5 k$	
			•3 interpret intercept	•3 $\log_5 k = 3$	
			•4 use laws of logs	•4 $k = 125$	
			•5 interpret gradient	•5 $n = -2$	

Question		Generic scheme	Illustrative scheme	Max mark
7.	(continued)			
		Method 4	**Method 4**	
		•1 interpret point on log graph	•1 $\log_5 x = 0$ and $\log_5 y = 3$	
		•2 convert from log to exponential form	•2 $x = 1, y = 5^3$	
		•3 interpret point and convert	•3 $\log_5 x = 2$ **and** $\log_5 y = -1$ $x = 5^2$ and $y = 5^{-1}$	
		•4 substitute into $y = kx^n$ and evaluate k	•4 $5^3 = k(1)^n \Rightarrow k = 125$	
		•5 substitute other point into $y = kx^n$ and evaluate n	•5 $5^{-1} = 5^3 \times 5^{2n}$ $\Rightarrow 3 + 2n = -1$ $\Rightarrow n = -2$	

Notes:

1. In any method, marks may only be awarded within a valid strategy using $y = kx^n$.
2. Markers must identify the method which best matches the candidates approach; markers must not mix and match between methods.
3. Penalise the omission of base 5 at most once in any method.
4. In Method 4, candidates may use $(2, -1)$ for •1 and •2 and $(0,3)$ for •3.
5. Do not accept $k = 5^3$.
6. In Method 3, do not accept $m = -2$ or $\text{gradient} = -2$ for •5.
7. Accept $y = 125x^{-2}$ for •5.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ determine expression for area of pond •² obtain expression for y •³ demonstrate result	•¹ $(x-3)(y-2)$ stated or implied by •³ •² $y=\frac{150}{x}$ •³ $A(x)=(x-3)\left(\frac{150}{x}-2\right)$ eg $A(x)=\frac{150x}{x}-\frac{450}{x}-2x+6$ $A(x)=156-2x-\frac{450}{x}$	3

Notes:

1. Accept any legitimate variations for the area of the pond in •¹, eg $A=150-2(x-3)-2(y)(1.5)$.
2. Do not penalise the omission of brackets at •¹. See Candidate A.
3. The substitution for y at •³ must be clearly shown for •³ to be available.

Commonly Observed Responses:

Candidate A		
$A(x)=x-3\times y-2$	•¹ ✓	
$A(x)=x-3\times\frac{150}{x}-2$	•² ✓	
$A(x)=156-2x-\frac{450}{x}$	•³ ^	

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)		•4 express A in differentiable form •5 differentiate •6 equate expression for derivative to 0 •7 solve for x •8 verify nature of stationary point •9 determine maximum area	•4 $156-2x-450x^{-1}$ stated or implied by •5 •5 $-2+450x^{-2}$ •6 $-2+450x^{-2}=0$ •7 $x=15$ •8 table of signs for derivative $\therefore$ maximum or $A''(x)=-900x^{-3}$ and $A''(15)<0$ $\therefore$ maximum •9 $A=96\left(\text{m}^2\right)$	6

Notes:

4. For a numerical approach award 0/6.
5. •6 can be awarded for $450x^{-2}=2$.
6. For candidates who integrate any term at the •5 stage, only •6 is available on follow through for setting their 'derivative' to 0.
7. •7, •8, and •9 are only available for working with a derivative which contains an index ≤ -2.
8. $\sqrt{\frac{450}{2}}$ must be simplified at •7 or •8 for •7 to be awarded.
9. Ignore the appearance of -15 at mark •7.
10. •8 is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 15.
11. •9 is still available in cases where a candidate's table of signs does not lead legitimately to a maximum at •8.
12. •8 and •9 are not available to candidates who state that the maximum exists at a negative value of x.

Question		Generic scheme	Illustrative scheme	Max mark
8.	(b)	**(continued)**		

Notes (continued)

For the table of signs for a derivative, accept:

x	15^-	15	15^+
$A'(x)$	$+$	0	$-$
Slope or shape	/	—	\

x	$\rightarrow$	15	$\rightarrow$
$A'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Arrow are taken to mean 'in the neighbourhood of'

x	a	15	b
$A'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Where $0<a<15$ and $b>15$

For the table of signs for a derivative, **do not** accept:

x	$\rightarrow$	-15	$\rightarrow$	15	$\rightarrow$
$A'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is discontinuous $-15 \rightarrow 15$ is not acceptable

x	a	-15	b	15	c
$A'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is discontinuous $-15<b<15$ is not acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $A'(x)$ are: A', $a'(x)$, $\frac{dA}{dx}$, and $-2+450x^{-2}$.

Commonly Observed Responses:

Candidate B - differentiating over multiple lines		**Candidate C - differentiating over multiple lines**	
	$\bullet^4$ ^	$A(x)=156-2x-450x^{-1}$	$\bullet^4$ ✓
$A'(x)=-2-450x^{-1}$		$A'(x)=-2-450x^{-1}$	
$A'(x)=-2+450x^{-2}$	$\bullet^5$ ✗	$A'(x)=-2+450x^{-2}$	$\bullet^5$ ✗
$-2+450x^{-2}=0$	$\bullet^6$ $\boxed{✓ 1}$	$-2+450x^{-2}=0$	$\bullet^6$ $\boxed{✓ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ substitute for y in equation of circle	•¹ $x^2+(3x+7)^2-4x-6(3x+7)-7=0$	5
			•² arrange in standard quadratic form	•² $10x^2+20x=0$	
			•³ factorise	•³ $10x(x+2)=0$	
				•⁴ •⁵	
			•⁴ state x coordinates	•⁴ 0 -2	
			•⁵ state corresponding y coordinates	•⁵ 7 1	

Notes:

1. •¹ is only available if '$=0$' appears by the •³ stage.
2. At •³, the quadratic must lead to two distinct real roots for •⁴ and •⁵ to be available.
3. At •³ do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
4. If a candidate arrives at an equation which is not a quadratic at •² stage, then •³, •⁴ and •⁵ are not available
5. •³ is available for substituting correctly into the quadratic formula.
6. •⁴ and •⁵ may be marked either horizontally or vertically.
7. Ignore incorrect labelling of P and Q.

Commonly Observed Responses:

Candidate A - substituting for y

$\left(\frac{y-7}{3}\right)^2+y^2-4\left(\frac{y-7}{3}\right)-6y-7=0$ •¹ ✓

$\frac{10y^2-80y+70}{9}=0$ •² ✓

$10(y-1)(y-7)=0$ •³ ✓

$y=1$ or $y=7$ •⁴ ✓

$x=-2$ or $x=0$ •⁵ ✓

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		•6 state centre of circle •7 calculate midpoint of PQ •8 calculate radius of small circle •9 state equation of small circle	•6 $(2, 3)$ •7 $(-1, 4)$ •8 $\sqrt{10}$ •9 $(x-2)^2+(y-3)^2=10$	4

Notes:

8. Evidence for •6 may appear in part (a).
9. Where a candidate uses coordinates for P and Q without supporting working, •7 is not available, however •8 and •9 may be awarded.
10. Where candidates find the equation of the larger circle •8 and •9 are not available.

Commonly Observed Responses:

Candidate B - using substitution Equation of smaller circle of form $(x-2)^2+(y-3)^2=r^2$ •6 ✓ Midpoint PQ $(-1, 4)$ •7 ✓ $(-1-2)^2+(4-3)^2=r^2$ $r^2=10$ •8 ✓ $(x-2)^2+(y-3)^2=10$ •9 ✓	**Candidate C - using tangency** Equation of smaller circle of form $(x-2)^2+(y-3)^2=r^2$ •6 ✓ Since $y=3x+7$ is tangent to smaller circle $10x^2+20x+20-r^2=0$ has equal roots $\Rightarrow 20^2-4(10)(20-r^2)=0$ •7 ✓ $\Rightarrow r^2=10$ •8 ✓ $(x-2)^2+(y-3)^2=10$ •9 ✓
Candidate D - using P or Q to mid-point as radius ⋮ $r=\sqrt{(-2+1)^2+(1-4)^2}=\sqrt{10}$ •8 ✗ or $r=\sqrt{(0+1)^2+(7-4)^2}=\sqrt{10}$ •8 ✗ $(x-2)^2+(y-3)^2=10$ •9 $\boxed{✓\ 2}$	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•[1] evaluate P for $t = 24.55$	•[1] 929	1

Notes:

1. Accept any answer which rounds 929.0368007… to at least 2 significant figures.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[2] substitute for P and D	•[2] $850 = 0.188807(600-210)^k$	4
			•[3] arrange equation in the form $a = b^k$	•[3] $\dfrac{850}{0.188807} = (600-210)^k$	
			•[4] write in logarithmic form	•[4] eg $\ln\left(\dfrac{850}{0.188807}\right) = \ln(600-210)^k$ or $k = \log_{(600-210)}\dfrac{850}{0.188807}$	
			•[5] solve for k	•[5] 1.41	

Notes:

2. •[3] may be implied by •[4].
3. Any base may be used at •[4] stage.
4. Accept 1.4 at •[5].
5. The calculation at •[5] must follow from the valid use of exponentials and logarithms at •[3] and •[4]. See Candidate A.
6. For candidates who take an iterative approach to arrive at the value $t = 1.41$ award 1/4. However, if, in the iterations P is calculated for $t = 1.405$ and $t = 1.415$ then award 4/4.

Commonly Observed Responses:

Candidate A - invalid use of exponentials		Candidate B - transcription error	
$850 = 0.188807(600-210)^k$	•[2] ✓	$850 = \underline{0.18807}(600-210)^k$	•[2] ✗
$850 = 73.63473^k$	•[3] ✗ •[4] ✗ •[5] ✗	$4519.59\ldots = 390^k$	•[3] $\boxed{✓ 1}$
$\log_{73.63473} 850 = k$		$\log_{390} 4519.59\ldots$	•[4] $\boxed{✓ 1}$
1.56…		1.41…	•[5] $\boxed{✓ 1}$

[END OF MARKING INSTRUCTIONS]